

# Damián Bravo Zamora on Guido Kreis's "Negative Dialektik des Unendlichen"

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By Damián Bravo Zamora

Any engagement with the history of philosophy that is worth the philosopher's while is irremediably philosophical. The spirit with which the philosopher approaches the history of his or her own activity is an incorrigibly Socratic one. Any discourse that smacks of an arbitrary appeal to authority, of an act of hiding behind the shield of unclarified concepts, or of a plotless chronology of authors, appears to the eyes of the philosopher not only as superfluous and insubstantial, but also, and above all, as intolerably boring. It is to Guido Kreis's great credit to have delivered a truly philosophical history, and even the story, of a philosophical problem. His book *Negative Dialektik des Unendlichen: Kant, Hegel, Cantor* is an ambitious and at the same time accomplished work, which squarely takes on a fundamental metaphysical and logico-mathematical problem by unhesitatingly dealing with the ideas at issue in an argumentative and unitary manner, reaching far beyond any specific exegetical concerns, of interest only to the specialist in the philosophical system of this or that thinker, and far beyond any specific technical difficulties appertaining to this or that area of contemporary philosophy.

The one problem which occupies the whole book is the fact that contradictions can be derived, and have been derived, from the assumption that there are absolutely comprehensive totalities, whether we take this assumption to mean that there is an individual *object* that has everything in it, or we shy away from this ontological commitment regarding the all-encompassing collection, and content ourselves with simply claiming that *there are all objects*. In both cases, Kreis's concern is with the concept of *everything*. "*Alles: das ist der begriffliche Kern der Paradoxien des Weltbegriffs*", says Kreis near the end of his book (p. 424). The reference to the concept *<world>* is quite pertinent here. For that concept will allow Kreis to start his philosophical tale with Kant, who attempted (unsuccessfully, according to Kreis) to derive a series of contradictions ('antinomies') from the concept *<spatiotemporal world>*, but who also acknowledged the existence of a 'transcendental' concept of the world, a concept through which we conceive the world simply as the totality of all existing things

(cf. A419/B447). Given that this transcendental concept of the world leaves it unspecified whether amongst 'all existing things' we should only count concrete or also abstract, only real or also imaginary, only actual or also possible 'things', the concept is fairly neutral to a myriad of ontological positions, i.e. to philosophical positions regarding *what there is*.

This transcendental concept will therefore be perfectly suited to relate Kant's concerns to a group of more radical problems, deriving from the work of Georg Cantor, regarding absolutely comprehensive totalities. In effect, as we shall see later on in this essay, a theorem proved by Cantor for mathematical *sets* of objects is incompatible with the assumption that there is an absolutely all-encompassing *set*—the set that has everything in it, the universal set. And, as Kreis forcefully argues—heavily drawing, in this case, on the work of Grim (1991) and Williamson (2003)—a similar strategy to that which Cantor used to prove his theorem could be used to state a series of rightfully called 'Cantorian' paradoxes for concepts like that of *all objects*, or *all facts* (and even *all truths* and *all propositions*), without assuming—and this is the important part—that there is a *set* of these things. The outcome is therefore that what seem to be the metaphysically most ambitious concepts of the world are contradictory concepts, i.e. concepts such that contradictions can be derived from the assumption that something 'out there' corresponds to them (either an individual object that has all objects or all facts in it as its members, or simply the genuine, no-individual-object-constituting multiplicities of all things or all facts).

But the purpose of Kreis's book is not merely that of stating paradoxes regarding absolutely infinite totalities. Throughout the book Kreis discusses the stances that philosophy should take and has taken towards the paradoxes, and towards itself *because* of the paradoxes. This explains Kreis's engagement with dialectics, and in particular with Hegel's *Science of Logic*. For, as Kreis points out, even though Kant clearly sees the antinomies as a dialectical performance of reason, the word 'dialectical' has a negative sense for Kant. Kant's purpose is to set the antinomies aside, and he therefore does not conceive his own philosophy as dialectical in nature. Kreis nonetheless calls Kant's philosophy 'limitative dialectics' because the outcome of the discussion of the antinomies is a limitation of our knowledge, which is reflected in Kant's doctrine of transcendental idealism.

Hegel's attitude towards the antinomies, and to dialectics in general, is quite different. Hegel sees the relevant contradictions as the motor of his own philosophical method. The final purpose is to *solve* the contradictions, but before that result every single fundamental concept, every category, has to be investigated as to whether it entails an antinomy or not. And the solution to the

paradoxes cannot involve a limitation of knowledge or thought, for only thought can draw the relevant limit, which means that thought cannot be determined by the limit. Hegel's philosophy, therefore, can properly be called 'positive dialectics', an explicit endorsement of the dialectical method as a method of knowing *what there is*, together with the view that this method eventually solves every contradiction that is contained in our concepts.

But Hegel's positive dialectics, according to Kreis, cannot be accomplished. The system's final statement, 'The concept is everything', cannot be considered definitive if the concept *<everything>* has not been investigated as to whether it entails a contradiction or not. But, alas, the Cantorian paradoxes teach us that the concept should have been thus investigated, for the concept *does* entail contradictions (p. 359). Given that, according to Kreis, no absolutely satisfactory solution to the antinomies is available (no solution that does not require that we give up general principles of thought), the proper philosophical stance towards the paradoxes, a philosophy whose main purpose is the realisation of such a conceptual stalemate, can receive the name of 'negative dialectics'. This is Kreis's own position.

In what follows I shall concentrate on Kreis's treatment of the Kantian antinomy and of the Cantorian paradoxes. An analysis of Kreis's highly interesting engagement with Hegel is beyond the scope of the present paper.

### **Kant on the Concept *<World>*, an Alleged Antinomy, and the Possible Ways Out**

Kant's antinomy of pure reason is the perplexing situation that reason finds equally convincing arguments for opposed statements about the world. The concept *<world>* here is that of the absolute totality of phenomena, by which is meant the absolute totality of spatiotemporal objects and events (A418–19/B446–7). The consideration of the world as a collection, but more specifically as a particular kind of *series*, is essential in this respect. For the contradictions are said to emerge when reason takes as a starting point an unproblematic object of experience and thinks of it as a *conditioned* object. By its own nature, reason asks for the *condition* to the conditioned object, arriving thus at the idea of a new object. But reason restates the question each time it arrives at a new object, and hence it is led to form the idea *<world>* as the absolutely comprehensive series of conditions to all spatiotemporal objects and events.

Kant presents four such antinomial conflicts, and they can be divided into two groups. The first group of antinomies emerges when one considers the world as a sum-total, a mathematical whole of things and events in space and time, subject to addition or aggregation of new elements, and also subject to division

(‘synthesis in the great as well as in the small’). According to Kant, then, we have equally convincing arguments for the claims that the world is finite in space and time, and that it is infinite in space and time (First Antinomy); the claims that every composite thing in the world consists of simple, indivisible parts, and that neither the world nor any composite thing in the world consists of simple parts (Second Antinomy). But ‘the very same world’ can be considered not merely mathematically but also ‘dynamically’, says Kant. This happens when we consider the world not as regards addition and division, but as regards ‘the unity in the existence of the appearances’. Thus we can regard the world as a series of *causal* conditions of spatiotemporal objects and events, and we can see objects as ontologically depending on, or as owing their existence to, other objects.

This dynamical conception of the world gives thus birth to the other two antinomies: we have equally convincing arguments for the claim that causality in accordance with the laws of nature is not the only kind of causality that obtains (the assumption of causality through freedom being necessary in order to explain some appearances) and for the claim that there is no freedom at all, but everything happens in accordance with the laws of nature (Third Antinomy); finally, reason is said to find equally good arguments for affirming that there is an absolutely necessary being and that there is no such absolutely necessary being (Fourth Antinomy).

It is fundamental for understanding this discussion to realise that Kant presents these conflicts as arising for those philosophers who do *not* endorse Kant’s doctrine of transcendental idealism. In other words, these conflicts arise for those who ‘conflate’ phenomena and things in themselves and think that those objects to which we have access through experience (phenomena) are also mind-independent objects (things in themselves). Kant’s doctrine has generated enormous controversy over the past two centuries, both exegetically, as regards the proper and accurate way to understand the various and ambiguous passages in which Kant presents his doctrine, and philosophically, regarding the question whether the doctrine is plausible on any (sympathetic) interpretation of it. Kreis is not indifferent to the controversy, and throughout his book he presents an interpretation of transcendental idealism that sees it as the semantic doctrine that statements about objects that in principle transcend all possible experience are statements which lack objective validity, meaning that such statements lack truth value.<sup>[1]</sup>

Seen in this light, Kant’s claim that the antinomies are unavoidable for transcendental realists means that realists cannot fail to endorse either the theses or the antitheses of the antinomial conflicts. Given that the theses and

antitheses are opposing claims about a particular object (the world), and that for realists such claims must have a truth value regardless of the fact that the object in question transcends all experience, then in principle there is no reason why the world should not be seen as the object that grounds either of the two opposing claims. The problem (antinomy) is, of course, that if the arguments that lead to the conclusions (theses and antitheses) are sound, then the world must be seen as grounding *both* of the opposing claims.

Given the larger context of the discussion that will occupy the whole book, Kreis's attention focuses on the first antinomial conflict. As Kreis himself points out, Kant's more metaphysical version of the concept *<world>* appears when he tells us that "in the transcendental sense the word 'world' signifies the absolute totality of the sum total of existing things" (A419/B447). As we said before, this description leaves open the possibility that as "existing things" our ontology accepts also non-spatiotemporal, non-concrete, i.e. abstract objects. Hence, this transcendental concept of the world could legitimately be interpreted as the concept that will prompt the Cantorian discussion regarding an absolutely comprehensive totality of all objects whatsoever. This, however, is *not* the concept that generates Kant's antinomies. Rather, the concept that generates the antinomies can be seen as a limitation of the previous, transcendental concept. What is at stake in the antinomies is the concept *<world>* as the sum-total of all *spatiotemporal* objects (and events, causes, conditions of existence, etc.). Now, particularly in the First Antinomy, reason is seen as iterating further and further spatial and temporal items to a series that constitutes the world-whole, hence as arriving at the concept of the absolutely comprehensive totality of these items. Hence, Kant's First Antinomy can be seen as a more circumscribed version of the discussion at a more radical level to come later in the book. This explains why Kreis's treatment focuses on the First Antinomy.

Here is a reconstruction of Kant's argument (in A426/B454) for the thesis of the first antinomy:

1. Assume that the world did not have a beginning in time (i.e. that the series of past events is infinite).
2. If so, then up to any given moment (up to, say, a second ago) an eternity has elapsed.
3. That up to any given moment an eternity has elapsed means that up to that given moment an infinite series of successive events has occurred, i.e. that an infinite series has been completed.

4. But an infinite series, by definition, can never be completed by means of a successive synthesis.
5. If we assume that the world did not have a beginning in time, then we arrive at a contradiction.
6. Therefore, it is false that the world did not have a beginning in time (i.e. that the series of past events is infinite).
7. Therefore, the world had a beginning in time (i.e. the series of past events is finite).

And here is a reconstruction of the argument (in A428/B457) for the antithesis:

1. Suppose that the world had a beginning in time.
2. That the world had a beginning means that it came into existence preceded by a time in which it (the world) *was not*.
3. But since we are speaking about the *world* (the *totality* of appearances), a time in which the world was not is an *empty time*. Thus if the world had a beginning, this means that it came into being in an empty time.
4. But no coming into being in an empty time is possible at all, because no part of an empty time possesses, as compared with any other, a distinguishing condition of existence rather than nonexistence.
5. Therefore, the world could not have had a beginning in time.
6. Therefore, the world is infinite with respect to past time.

As can be seen, the argument for the thesis rests heavily on what we identified as step 4, the definition of an infinite series as one that cannot be completed. The argument for the antithesis, on the other hand, rests on Kant's version of the Leibnizian principle of sufficient reason (step 4) together with the assumption that a beginning of the world (thought of as the world's first event) must be preceded by an empty time (step 2).

Kreis argues—correctly, in my opinion—that Kant fails to demonstrate the existence of a genuine antinomy in the concept *<world>*. Even though Kreis presents an interesting discussion of the antithesis, claiming that it could at least be rendered plausible by interpreting it in the light of the law of the conservation of matter (pp. 64–76), the crucial point is that the definition of an infinite series, on which the argument for the thesis heavily rests, is quite clearly a definition

only a finitist could accept. The argument is therefore circular. But if this is so then there is no ground for accepting the thesis, and hence none for accepting the existence of the whole antinomy either.

These objections, however, far from legitimising any attempt at disregarding Kant's engagement with the concept *<world>*, only serve to bring the discussion to a more fertile ground. The interesting questions emerge when we consider the hypothetical case in which Kant had succeeded in demonstrating the existence of a genuine antinomy. According to Kreis, Kant develops three related strategies to counter the emergence of the purported antinomy. They are tantamount to a diagnosis of their dialectical generation, and they provide therefore the basis for a solution of the conflict.

Kant's first and main strategy, according to Kreis, consists in describing the concept *<world>* as an empty concept, in the sense that it is the concept of something that cannot be given (either directly or indirectly) as the object of an empirical intuition.<sup>[2]</sup> This realisation is, in fact, a first step in Kant's argumentation, as presented by Kreis. The sheer fact that the purported object (the world) cannot be given in an intuition does not directly justify us in affirming the emptiness of its concept. The transcendental realist could take up God's perspective, and hence allow the possibility of there being an object that corresponds to the concept *<world>*. God's perspective, however, is precisely what we do not have, according to Kant. We cannot even take for granted that the concept of such a perspective, or the statements that can only be made from such a perspective, make any sense. In fact, Kant has an 'intuition-criterion' for non-empty concepts, says Kreis, and this criterion is precisely what is not fulfilled by the metaphysical concept of the world. This is how Kreis explains Kant's position:

Deshalb ist der metaphysische Begriff der Welt nach Kant ein prinzipiell leerer Begriff. Er unterscheidet sich damit von Begriffen, die nur **kontingenterweise** leer sind, weil es faktisch keinen Gegenstand gibt, der unter sie fällt (der Begriff **direkter Nachkomme von Immanuel Kant** etwa ist faktisch leer, müßte es aber nicht sein). Der metaphysische Begriff der Welt unterscheidet sich aber auch von widersprüchlichen Begriffen wie **rundes Viereck**, die aus logischen Gründen notwendigerweise leer sind, weil ihre Instantiierung logischen Prinzipien widerspräche. Der metaphysische Begriff der Welt ist vielmehr aus transzendentalphilosophischen Gründen notwendigerweise leer, weil seine Instantiierung dem Postulat der (direkten oder indirekten) Veranschaulichung widerspräche. Wenn aber der Begriff der Welt ein **per definitionem** und aus transzendentalphilosophischen Gründen notwendig leerer Begriff ist, dann sind auch alle Aussagen über die Welt notwendigerweise leere Aussagen:

- (1) Die Welt ist unendlich.
- (2) Die Welt ist nicht unendlich.
- (3) Die Welt existiert.
- (4) Die Welt existiert nicht.

Unter diesen Voraussetzungen kann die Antinomie nicht entstehen. (pp. 123–4)

Kant's first strategy is, therefore, intimately related to transcendental idealism. So are the two other strategies identified by Kreis. One of them involves charging the transcendental realist with a fallacy of equivocation with respect to the concept *<the conditioned>*: it is understood as the concept of an empirical object (hence as a phenomenon) when it is seen as unproblematically given, but it is understood as a thing in general (hence as a thing in itself) when from the fact that a conditioned object is given it is inferred that the absolutely unconditioned series of its conditions must also be given (pp. 127–30). The third strategy involves Kant's distinction (A504–5/B532–3) between dialectical and analytical opposites. The antinomy only arises because the transcendental realist is unaware of the fact that the two opposed statements which constitute the antinomy ('The world is finite' and 'The world is infinite') make the unjustified presupposition that the object in question (the spatiotemporal world) exists in itself. Making this presupposition, the opposition seems exhaustive, as the principle of contradiction would permit only one of the opposed statements. But if

one adopts transcendental idealism, one is entitled to refrain from either statement. For in this case one can see the opposition in question, when properly reconstructed, as constituted by predicate-negations instead of statement-negations. The finitist's claim would then be 'The world is non-infinite' instead of 'It is not the case that the world is infinite'. Similarly for the infinitist position. The claim would be 'The world is non-finite' instead of 'It is not the case that the world is finite'. These reformulations make it clear that there is an existence assumption on both sides of the antinomy. And given that transcendental idealism teaches that the object to whose existence both finitists and infinitists are committed transcends all possible experience, and hence that all statements regarding the purported object lack objective validity, the antinomy is immediately defused on account of its emptiness (cf. pp. 130–7).

Up to this point, Kreis's account of Kant's strategies for solving the antinomial conflict can be considered more or less 'official'. That the main strategy and also the derivative ones are in one way or another dependent on the truth of transcendental idealism (even under the non-ontological, rather semantical and metalinguistic way in which Kreis interprets this doctrine) should not come as a surprise, for Kant himself sees in the first two, mathematical antinomies an indirect proof of transcendental idealism, "if perhaps someone did not have enough in the direct proof in the Transcendental Aesthetic" (A506/B534). Under the assumption of transcendental realism the antinomy is unavoidable; reject transcendental realism, hence assume the truth of transcendental idealism, and the antinomy disappears. This is Kant's own line of thought.

Kreis advances a relatively bolder exegesis when it comes to what he calls 'Kant's constructive proposal' for the solution of the antinomy. One first moment of this constructive proposal would be carried out by the substitution of an empirical concept of the world for a metaphysical one, which goes hand in hand with the substitution of the concept <world> as a *comparative* whole for the one that would have generated the antinomies, the concept <absolute whole>. Here is an important passage on which Kreis bases his interpretation:

The whole, in an empirical signification, is always only comparative. The absolute whole of magnitude (the world-whole), of division, of descent, of the conditions of existence in general, together with all the questions about whether these are to come about through a finite or an endlessly continuing synthesis, has nothing to do with any possible experience. [...] But it is really this whole for which an explanation is being demanded in the transcendental problems of reason. (A483–4/B511–12)

Kant is indeed drawing a distinction between two concepts of the world, a comparative and an absolute one. And Kant is also stating unequivocally that it is the latter one which generates the "problems of reason", i.e. the antinomies. But Kreis also thinks that Kant is here fleshing out the comparative, hence empirical concept of the world, with which we shall be able to escape the antinomy. According to Kreis, the context suggests that Kant could have had the following definition of the world in mind:

(W2\*) Die Welt ist die Reihe aller bislang bekannten Ursachen aller bislang bekannten Zustände aller bislang bekannten raumzeitlichen Gegenstände.

Zu diesem empirischen kosmologischen Weltbegriff läßt sich problemlos ein analoger semantischer Weltbegriff bilden:

(W1\*) Die Welt ist die relative Totalität aller bislang bekannten Gegenstände. (p. 138)

The emphasis, of course, should be on the "*bislang bekannten*" part of the present concepts. The introduction of this specification immediately turns the concept <world> into one that is acceptable in the eyes of the transcendental idealist, a non-empty concept of the empirical world. But it also has a number of surprising consequences. Kreis expresses some of these consequences in the following way:

Die empirische Welt ist ein **relativ** zu unserer Erkenntnis **dynamischer** Gegenstand; kein ein für allemal statisch feststehender Behälter für alles, was es in der Welt gibt, sondern ein im Laufe unserer naturwissenschaftlichen Forschungen stetig »anwachsender«, also den in ihm erforschten Kausalverhältnissen nach immer reichhaltigerer Gegenstand und in diesem Sinne ein **komparatives** All. (p. 139)

The metaphor of an "*anwachsender*" object points towards a more Aristotelian conception of the world as a potentially infinite, but never actually given whole. And this is a conclusion that Kreis will in fact draw. But first he tells us what the expressions "*relativ zu unserer Erkenntnis*" and "*bislang bekannten*" mean in this respect:

Wenn wir im Rahmen unserer (naturwissenschaftlichen) Theorien Dinge beschreiben, die uns zuvor unbekannt waren, dann können wir diesen Vorgang so interpretieren, daß sie in unsere Welt hineingeholt werden. Wir werden sie **dann** auch als Dinge behandeln, die es in unserer Welt **zuvor** bereits gegeben haben muß, aber diese Zuschreibung erhalten sie erst ex post, nachdem sie in unsere Welt hereingeholt worden sind, wir sie kennengelernt haben und insofern unsere Erkenntnis der empirischen Welt reichhaltiger geworden ist.

Kants empirischer Weltbegriff ist der Begriff einer potentiell unendlich anwachsenden Anzahl von Gegenständen: Zu jeder bestimmten Gesamtheit von bislang bekannten Gegenständen, die zur Extension von **Welt** gehören, läßt sich stets wieder ein weiterer Gegenstand finden, der auch zur Welt gehört, aber in der jeweiligen bestimmten Gesamtheit noch nicht enthalten war. (p. 140)

Through new empirical discoveries of objects, therefore, our concept of the world becomes a more inclusive one. It includes these new entities. It includes them as objects that already existed, but the fact that they were not considered in the previously determined concept of the world is sufficient for Kreis to say that the totality in question can be considered a 'growing' one.

It is this aspect of Kreis's interpretation of Kant that seems to me textually and philosophically unwarranted. I shall argue for this in a moment, but first let me point to the fact that this purported substitution of a comparative for an absolute concept of the world is not a minor aspect of Kreis's interpretation of Kant, for it is here that Kreis will find a devastating problem for Kant. An alleged result of this Kantian substitution of concepts is the fact that

wir uns immer nur auf eine **relative** Totalität aller Erscheinungen widerspruchsfrei und objektiv gültig beziehen können, nie aber auf die **absolute** Totalität aller Erscheinungen. [...] Die Ersetzung des Weltbegriffs [...] macht die universale Quantifizierung über uneingeschränkt alle Gegenstände unmöglich. Der Geltungsanspruch der Grundsätze des reinen Verstandes setzt aber genau diese universale Quantifizierung über uneingeschränkt alle Gegenstände voraus. (p. 146)

In other words, the limitation of our cognition to the world understood as a comparative whole goes against the strongly *a priori* nature of the principles that Kant defends in the Analytic. Kant's 'constructive proposal' of a concept

substitution turns out to be in fact self-refuting when considered in relation to Kant's own doctrines of the nature and possibility of *a priori* knowledge.

In my opinion, however, no such devastating conclusions need to be drawn. Kant could not possibly have held that the correct concept of the world is that of the world as a comparative whole, if by this concept is meant the totality of all "*bislang bekannten*" objects. No context in which Kant argues in favour of an empirical and even comparative concept of the world suggests this interpretation. Apart from the previously quoted one, the only other passage on which Kreis bases his interpretation of the comparative concept of the world as the totality of what is known so far is the following one:

That there could be inhabitants of the moon, even though no human being has ever perceived them, must of course be admitted; but this means only that in the possible progress of experience we could encounter them; for everything is actual that stands in one context with a perception in accordance with the laws of the empirical progression. Thus they are real when they stand in an empirical connection with my real consciousness, although they are not real in themselves, i.e., outside this progress of experience. (A492–3/B521).

Even though Kant is indeed describing the real as that which relates to our perception, and hence one could try to find here a textual justification for Kreis's comparative concept of the world as the concept of the totality of all objects 'known so far', such an attempt is invalidated by Kant's immediate addition "in accordance with the laws of the empirical progression". These laws are, as we know, the principles of the understanding—precisely the ones that, on Kreis's reading, are rendered invalid by the endorsement of the comparative concept of the world. Kant is concerned here with the progress of *possible* experience, and those principles defended in the *Analytic* make a claim for universal validity precisely in this domain: the domain of possible experience. This domain must include all that is *empirically possible* (or, to echo Kant's passage, all that *possibly* stands in connection with perception), and hence the domain includes much more than what is 'known so far': it must include also all that *may* be empirically known. (What is in the future is part of what is possible.) Kant is not saying anything that should sound like:

To be (an empirical object) is to be perceived or to have been perceived.

He is saying something much closer to:

To be (an empirical object) is to *be able* to be perceived.

And the main motivation of the Analytic is to argue that not everything can enter this realm of *possible* experience.

Where does this leave Kant's own concept of the world and hence his more 'constructive' proposal for a solution to the antinomies? Kant's *own*, official concept of the world would undoubtedly have to be interpreted in the light of transcendental idealism. The realm of possible experience is the realm of phenomena, as opposed to that of things in themselves. *This* realm, i.e. the phenomenal world, must be thought of as correlative to empirical investigation, and hence as an ever-growing, potentially infinite realm. But note that Kant need have no qualms about this conception of the world as potentially infinite. The realm of possibilities *within experience* is conceived by Kant as governed by the "laws of empirical progress".

This is not to say that Kant's official concept of the world is unproblematic, let alone that Kant's arguments in the Analytic succeed in defending the view of the realm of possible experience as law-governed (although I *am* claiming that Kant's concept is not beset by the self-refuting problems that Kreis attempts to derive from it). Moreover, in my view Kant's official concept of the world, interpreted, as it should be, in the light of transcendental idealism, cannot be justified in virtue of the discussion of the antinomies—in spite of Kant's claim that the (mathematical) antinomies provide an indirect proof of transcendental idealism. This can be seen by considering a possible way out of the antinomies that Kant apparently did not consider in its full significance, and to which Kreis does not pay too much attention. One of the few commentators that succeed in pointing to this flaw in Kant's argumentation in a clear and perplexingly stark manner is Moltke Gram. He writes:

From [the Antinomy] it follows that the referent [of the expression 'the totality of appearances'] is nonexistent. Now, if this is the real structure of Kant's argument, then he has proved that the totality of appearances does not exist as a thing in itself; for he has established that it does not exist at all, and a fortiori it does not exist in itself. But this conclusion has nothing to do with the characteristics peculiar to things in themselves. Nor does it turn on the notion of transcendental ideality. It is an immediate inference from the conclusion—which Kant's argument does establish—that the object called the totality of appearances does not exist at all. (Gram 1967:512)

What Gram is saying is that, granting that Kant succeeds in deriving a genuine antinomy from the concept *<spatiotemporal world>*, then the only thing that follows is that the concept *<world>* is self-contradictory and therefore that no

object could possibly correspond to it. In other words, the concept <world>, in spite of its initially seeming to be unproblematic, turns out to be every bit as contradictory as that of <round square>. This certainly entitles Kant to his critique of the hypostatisation of the concept <world>, but not to his claim that he has provided an indirect proof of transcendental idealism, nor to his corresponding official interpretation of the concept <world> in the light of transcendental idealism.<sup>[3]</sup>

Kreis would probably disagree with my claim that the mere rejection of world-hypostatisation (*without* the endorsement of transcendental idealism) is a possible way out of the antinomy. But, as we saw in a previously quoted passage from pp. 123–4, in Kreis's view the transcendental realist must hold that one of these two sentences is true:

(3) The world exists.

(4) The world does not exist.

Both Kant and Kreis see the realist as endorsing (3), and hence as getting involved in the antinomy. But both Kant and Kreis in fact fail to consider (4) as a serious option for the realist—they do not even consider it as a post-antinomial option. In other words, even under the extremely generous assumption that Kant had succeeded in deriving an antinomy from the concept <world>, why could the realist not just accept that she was wrong in affirming (3), and instead affirm (4)? That would be a way for her to escape the antinomy without endorsing transcendental idealism. The world (if she decides to continue using the word 'world'), would be for the (post-antinomy) realist a genuine, no-individual-object-constituting multiplicity of objects and events.

Kreis might argue at this point that such an iconoclastic rejection-of-world-reification-full-stop! could not be taken seriously by Kant, either as an option for himself (transcendental idealist), or as an option for his opponent (transcendental realist). The idea <world> was seen by the philosopher of Königsberg as an *indispensable* one. In effect, as Kreis correctly points out (pp. 37–41; 141–3), this is the second moment of Kant's 'constructive' conception of the world and of his proposal for a solution of the antinomies. The concept <world> is seen as indispensable for accomplishing a task that reason sees as its own distinctive mission: to give unity and systematicity to our knowledge. Because of the antinomies, Kant rejects world-hypostatisation; but, because of the concept's indispensability, Kant is interested in retaining the concept, and hence recommends "einen nicht-metaphysischen Gebrauch eines metaphysischen Begriffs" (Kreis, p. 142): a regulative as opposed to a constitutive use of the concept <world>.

The problem, of course, with putting things this way is that it then it becomes clear that there is no reason why only transcendental idealists would have the privilege of taking the concept <world> as a merely regulative and non-constitutive one. The option of refraining from a fully-fledged hypostatisation of the concept <world>, of using the idea only for systematic purposes, of shying away from a commitment to the actual existence of the world, is as available to the realist as it is to the idealist.<sup>[4]</sup> Either we genuinely give the realist the option of denying the world's existence (and saying nothing more, resting content with the rejection of world-reification), or we cling to the view that the concept <world> is absolutely indispensable for everyone involved (i.e. for both realists and idealists), but then we are not justified in forbidding the realist to make a merely regulative use of the concept.

### **Cantorian Paradoxes, Solution Strategies, and the Prices We Are Ready to Pay**

Taking as a cue Kant's definition of the transcendental concept of the world, according to which the world is the totality of existing things (A419/B447), Kreis goes on to analyse some of the most interesting connections between the Kantian antinomies and the Cantorian paradoxes regarding absolutely comprehensive totalities. Given its importance for the rest of the discussion, let us briefly state the essentials of one of these paradoxes, namely, the so-called Cantor's paradox.

According to a theorem proved by Cantor, given any set  $\alpha$ , there is a more comprehensive set, namely the Power-set of  $\alpha$  (abbreviated  $P\alpha$ ).  $P\alpha$  is the set that has as members exactly the subsets of the original set  $\alpha$ . (A set  $\beta$  is a subset of  $\alpha$  if and only if, for any  $x$  that is a member of  $\beta$ ,  $x$  is also a member of  $\alpha$ .) Cantor's proof of this theorem consists in deriving a contradiction from the assumption that  $\alpha$  and  $P\alpha$  are the same size (i.e. have the same number of members, or, slightly more technically, have the same 'cardinality'). The criterion of equinumerosity between two sets is the existence of a one-to-one correspondence between the members of one set and the members of the other. Cantor's proof of his theorem consisted therefore in the derivation of a contradiction from the assumption that there is a one-to-one correspondence between the members of any given set  $\alpha$  and its Power-set  $P\alpha$ . Let us, then, assume that there is such a 'bijection' (the more technical word for a one-to-one correspondence) between  $\alpha$  and  $P\alpha$ . Let us call this bijection 'f'. Now consider the set  $D$  of all those members of  $\alpha$  that are not members of that member of  $P\alpha$  to which they are correlated by f. In set theoretical symbolism:  $D = \{x \mid (x \in \alpha) \ \& \ (x \notin f(x))\}$ . Now,  $D$  is a subset of  $\alpha$ .  $D$  must therefore be a member of  $P\alpha$ .  $D$  must therefore be correlated with exactly one member  $w$  of  $\alpha$ .

But here we get a contradiction. For one of these two possibilities must be the case: either  $w$  is a member of  $D$  or  $w$  it is not a member of  $D$ . Suppose it is. But  $D$  is the set constituted by exactly those members of  $\alpha$  that are *not* members of the member of  $P\alpha$  that  $f$  assigns to them. So  $w$  is not a member of its assigned member of  $P\alpha$ , and hence not a member of  $D$ . Suppose  $w$  is not a member of  $D$ . But then  $w$  fulfils the two conditions that define all and only the members of  $D$ : being a member of  $\alpha$ , and not being a member of the member of  $P\alpha$  that  $f$  assigns to it. So  $w$  is a member of  $D$ . This is the contradiction:  $w$  is a member of  $D$  if and only if it isn't. This contradiction has been derived from the assumption that  $\alpha$  and  $P\alpha$  are equinumerous, and thus, by *reduction ad absurdum* we can now conclude Cantor's theorem: Given any set  $\alpha$ , there is a more comprehensive set than  $\alpha$ , namely  $P\alpha$ .

A direct consequence of this theorem is that there cannot be a set of all sets. For suppose that there is a set of all sets, and call it 'U'. But  $PU$ , according to Cantor's theorem, has more elements than U. But note that  $PU$ 's members are sets (namely, the subsets of U), and thus  $PU$  is a set that has more sets as members than the set that supposedly had all sets. Contradiction. This is known as Cantor's paradox.

But there are other fairly direct consequences of Cantor's theorem that immediately touch upon our present concerns. For let us entertain the concept *<sum total of absolutely all things>* (whether concrete or abstract, spatiotemporal or non-spatiotemporal, actual or merely possible, etc.). If we think of this sum total as a Cantorian set, we get a contradiction. For call again 'U' the set of absolutely all things.  $PU$  has more members than U. The members of  $PU$  being also things,  $PU$  is a set that has more things than the set that supposedly had absolutely all things. Contradiction.

Now consider the concept *<sum total of absolutely all facts>*. If we think of this sum total as a Cantorian set, Cantor's theorem will again generate a contradiction. Call again 'U' the set of absolutely all facts. Now,  $PU$  has more members than U. But to any member of U there corresponds *at the very least* one fact: for example, the fact that it has exactly the members that it has. So there are more facts than those that are contained in U, which was supposed to be the set of absolutely all facts. Contradiction.

As my formulation of these paradoxes has made it clear, these are problems we are bound to get involved in whenever we think of the totalities in question (the sum total of all sets, the sum total of all things, the sum total of all facts) as *Cantorian sets*. I shall discuss in a second whether this is a justified assumption, but for the moment let me follow Kreis's lead and concentrate on a relatively

more recent discussion. This discussion turns around the claim that Cantorian problems emerge even when some of the totalities in question (in particular, the sum total of all things and the sum total of all facts) are not thought of as Cantorian sets. Cantorian paradoxes, we are told, are generated simply for the concepts *<all things>* and *<all facts>*, without assuming that there is a set that has as members all things or all facts.

These paradoxes are rightly called 'Cantorian' because the strategy for stating them parallels the one developed by Cantor for establishing his theorem: in particular, a contradiction will emerge from the assumption of a certain one-to-one correspondence, and from the realisation of the fact that one of the items of the allegedly correlated totalities cannot be thus correlated.

Kreis himself provides some very neat formulations of some of these paradoxes. In one of them, Kreis elaborates on work by Timothy Williamson.<sup>[5]</sup> In order to understand the statement of this contradiction, we have to take into account the following semantical considerations. Universal quantification, we might assume, is at least *prima facie* a desideratum of some formal languages. For example, if we want to express logical laws in a formal language, it is at least *prima facie* legitimate to assume that all objects and states of affairs whatsoever will conform to those laws. A legitimate question is therefore whether, according to an interpretation  $I_1$  of a formal language  $L$  that claims universal validity, a given predicate  $F$  applies or fails to apply to any given object  $x$ . Given that interpretations are also objects, we can ask whether  $F$  applies to the interpretation  $I_1$  itself or not. But note that *this* question can be answered either by  $I_1$  itself or by another interpretation  $I_2$  that also claims universal validity.  $I_1$  can answer this question negatively and  $I_2$  positively, and vice versa, or both positively, or both negatively.

But now think of that interpretation  $\mathfrak{D}$  (which also claims universal validity) according to which  $F$  applies to exactly those interpretations (with universal validity) under which  $F$  does *not* apply to the interpretations themselves. Does  $F$  apply to  $\mathfrak{D}$  or not? It applies to  $\mathfrak{D}$  if and only if it does not. Contradiction. Taking this paradox as a basis, Kreis formulates the following one, which closely parallels the previously stated ones in that it derives a contradiction from the assumption of a bijection, but which does not have recourse to the concept of a Cantorian *set* that would be the Universe that has all objects as members, but only to the assumption that we can talk about *all objects*:

Wenn es uneingeschränkt alle Gegenstände gäbe, dann müßte es eine Identitätsabbildung aller Gegenstände auf sich selbst geben. Angenommen also, das sei so. Wir können zwei Listen anfertigen: Auf der ersten stehen alle Gegenstände, auf der zweiten alle Gegenstände, mit denen sie identisch sind, also sie selbst. Unter allen Gegenständen sind auch die Interpretationen formaler Sprachen. Manche Gegenstände auf der zweiten Liste interpretieren ein beliebiges Prädikat  $F$  der von ihnen interpretierten Sprache daraufhin, ob es auf sie selbst zutrifft oder nicht. Nehmen wir diejenige Interpretation  $\mathfrak{D}$ , unter der das Prädikat  $F$  auf genau diejenigen Interpretationen zutrifft, unter denen  $F$  jeweils nicht auf diese Interpretationen selbst zutrifft. Da auf der zweiten Liste alle Gegenstände stehen, muß dort auch  $\mathfrak{D}$  stehen. Dann muß es aber auch einen Gegenstand auf der ersten Liste geben, von dem  $\mathfrak{D}$  der Zuordnungswert ist. Aber es kann ihn nicht geben, weil das Prädikat  $F$  auf ihn genau dann zutrifft, wenn es nicht auf ihn zutrifft. Also kann es keine Identitätsabbildung aller Gegenstände auf sich selbst geben. (p. 433)

Surely, the concept *<object>* is understood here in the widest possible sense, that in which also an interpretation of a formal language can be said to be an object. But there is no *a priori* reason why this should not be the case. And even though the concept of *bijection* (*Identitätsabbildung*), i.e. of a *one-to-one correspondence*, is used here, it is not used as a criterion of size-comparison between sets, as in set theory.

But this widest possible sense of the word 'object' allows us to generate a contradiction regarding the concept *<all facts>*, only now without the assumption that there is a set of all facts. To see this, we must take into account that facts *are about* or *concern* objects. The fact that my sweater is green is a fact about my sweater, and the fact that the Cathedral of Cologne is in Gothic style is a fact about the Cathedral of Cologne. Now some facts are about themselves and some are not. The fact that my sweater is green is about my sweater, not about itself, but the fact that all facts have predicative structure is a fact about all facts and thus also a fact about itself. Think now of all the facts that are not about themselves and about the fact—call it 'fact  $\mathfrak{D}$ '—*that* those facts are not about themselves. If there were all facts, we could build an identity bijection from all facts to themselves. But then fact  $\mathfrak{D}$  in the second 'list' would have to be correlated with a fact  $w$  in the first 'list', whilst fact  $w$  is no other than fact  $\mathfrak{D}$  itself. But is fact  $\mathfrak{D}$  about itself? It can easily be seen that it is if and only if it isn't. Contradiction.

Let us finally consider the following paradox regarding the concept *<all propositions>*.<sup>[6]</sup> Let us suppose that this concept refers to absolutely all propositions. If this concept is a legitimate one, it may appear in a proposition which expresses a fact concerning all propositions. Let us call '**P**' this purported proposition, and let us say that **P** is *about* all propositions in the sense that it either predicates or attributes something to all propositions or expresses that all propositions are related in some way to some entity or entities. So **P** is a proposition about all propositions. Let us call all propositions **P** is about (i.e. all propositions) **P**-propositions. So there is at least one way of correlating all **P**-propositions and all propositions *simpliciter*, namely, the bijection of *identity*. Consider now all **P**-propositions. Each and every one of them either *is* or *is not* about the proposition to which it is correlated by bijection. (In other words, each and every one of the **P**-propositions either *is* or *is not* about itself).

Think now of all those **P**-propositions that are *not* about the proposition to which they are correlated by bijection. (In other words, think of all the **P**-propositions that are not about themselves). Surely, there can be a proposition about all these propositions, namely, the proposition that these propositions are not about their correlated propositions. Call this latter proposition '**D**'. Since **D** is a proposition and **P** is about all propositions, **D** must be one of the **P**-propositions. It must therefore be correlated to a proposition, namely, to itself. Consider now the question whether **D** is or is not about itself. Suppose it is. But, by definition, **D** is about all those **P**-propositions which are not about themselves. So it is not about itself. But **D** is about all and only those **P**-propositions that are not about themselves, so, given that **D** is a **P**-proposition and that is not about itself, it must be about itself! So **D** is about itself if and only if it is not about itself. Contradiction.

The previous enumeration of Cantorian problems and paradoxes should enable us to discuss Kreis's work. The main purpose of this work is to defend the claim that these totalities generate serious contradictions, and that we are devoid of any strategy for solving these paradoxes that does not imply giving up cherished general principles of thought. Kreis himself points out that we should ponder which of these solution strategies implies the least of all possible evils. My impression, however, is that some of those 'evils' are less weighty than Kreis seems to think.

Take as an example Kreis's discussion of the so-called Cantor's Domain Principle. According to this principle, any "variable quantity in the sense of the potentially infinite" presupposes a corresponding actual infinity, which is the fixed domain of variability of the potential infinity.<sup>[7]</sup> This "fixed domain" is, of course, a Cantorian set. What Cantor has in mind is the following. Mathematical theories are made of propositions which use both free and bound variables. But the sense

of those propositions is not fixed unless the domain of variability of the variables is fixed.<sup>[8]</sup> If the relevant mathematical theories are not to be devoid of “each fixed support for the study”, then the domain of variability of their variables must be fixed, which is just another way of saying that that domain must be thought of as a Cantorian set.

Why is this principle relevant in the present context? We just saw that the first group of paradoxes involved thinking of the totalities in question (that of all things, and that of all facts) as Cantorian sets. A traditional and fairly mainstream strategy for solving the paradoxes has accordingly been to deny that those totalities are sets. For example, one might propose to think of the universe of sets as a potentially infinite, ever-growing collection of objects, and therefore *not* as a Cantorian set.<sup>[9]</sup> But if the Domain Principle applies at this general level also (i.e. not only requiring a fixed domain for the variables of the mathematical theories for which set theory provides a foundation, but also requiring a fixed domain for the variables of set theory itself and for any variables in any theory), then *there must be* the sets in question (all things, all facts). Were we to challenge the universal, unrestricted validity (if at all) of Cantor's Domain Principle,<sup>[10]</sup> Kreis would immediately present the following objection:

Die Konsequenz dieser Lösung besteht dann darin, jegliche Wissensansprüche in bezug auf die absolut unendlich großen Mengen an andere, nicht-mathematische Disziplinen abzugeben; für Cantor selbst kam hier nur noch die Theologie in Frage. Jede Option in der Richtung dieses Lösungsvorschlags hat aber zwangsläufig zur Folge, daß die mathematische Mengentheorie ihre methodische Souveränität einbüßt; sie führt **selbst** Unterscheidungen ein, die sie **selbst** an **andere** Disziplinen delegieren muß, und sie kann sich daher—ihrem eigenen Grundlagenanspruch ganz entgegengesetzt—nicht als autonome Disziplin selbst rechtfertigen. (p. 390)

But note that it is perfectly possible to think of the “*Grundlagenanspruch*” of set theory as the statement that set theory should clarify and construct the concepts of objects and operations of the other mathematical theories—not of set theory *itself*. Whether any foundation must be an absolutely unconditioned foundation is a philosophical thesis that needs justification. (That at some points reasons come to an end does not necessarily mean that there is no legitimate use of the word ‘reason’). Surely, Cantor's own conception of providing a foundation for some mathematical theories (“*mathematische Betrachtungen*”) was never absolutist to such an extent, for otherwise he would not have accepted the existence of genuine multiplicities that cannot consistently be thought of as ‘ones’. Ironically, Cantor himself never applied ‘Cantor's Domain Principle’ unrestrictedly. The myth

that every multiplicity constitutes a set (and hence an individual object) was not even attractive to the founder of set theory. Cantor was a collection-pluralist, to whom the 'sovereignty' of the discipline he was in the midst of founding did not seem to worry too much. Neither, I submit, should it worry *us* at all, at least as long as we lack a justification for our attempt to see every collection as a set.

A related case, in which Kreis thinks that the proposed solution-strategy comes at a high price, is Nicholas Rescher and Patrick Grim's attempt at conceptualising absolutely comprehensive totalities not as sets, but as other kind of collections. Rescher and Grim's is an explicit attempt at providing a paradox-free conception of the totalities with which Kreis is concerned throughout his book: the totality of all things, that of all facts, all propositions, etc. (Rescher & Grim 2011:73). And they do this by proposing the existence of non-set collectivities they call "plena":

A **plenum** as here understood is a collectivity that contains distinct entities corresponding to each of its sub-collectivities, where sub-collectivities follow the same pattern as subsets:

$$(\forall s) (s \subset P \leftrightarrow (\forall x) (x \in s \rightarrow x \in P))$$

The mark of **plena** is that **every sub-collectivity *s* of a plenum *P* is such that there is a member of *P* that exists in unique correlation with *s*.**

(Rescher & Grim 2011:61)

This "mark" of plena is in fact their defining characteristic: that there is a one-to-one correspondence between their members and their 'subplena'. Moreover, this one-to-one correspondence can be that of identity. In that case, Rescher and Grim talk about a "membership plenum" (2011:62). A membership plenum, in other words, *has* as members each of its subcollectivities, and is therefore self-membered.

The decisive point of Rescher and Grim's work is that the object that plays the role of the "diagonalized out" subcollection which could not be put in a one-to-one correspondence with any object of the original collection when we were talking about Cantorian sets, and which therefore provided the crux of Cantor's proof of his theorem (and of all the paradoxes we have been dealing with so far), is now characterised as an "indeterminate collectivity". By an indeterminate collectivity Rescher and Grim mean a collection *C* such that it is not determined, for every object *x*, whether *x* belongs to the collectivity *C* or not. And this is intended not as a mere "oscillation" in truth-value of the proposition ' $x \in C$ ', but as a *lack* or "full gap" in truth value (Rescher & Grim 2011:81). This involves,

indeed, a rejection of classical logic, in particular a rejection of the Law of Excluded Middle. In some cases, like the ones that create the paradoxes, neither ' $x \in C$ ' nor its negation ' $x \notin C$ ' have a truth-value, according to Grim and Rescher (2011:69–70, 73). But then we have no means of stating a parallel 'Cantorian' contradiction on the assumption of a one-to-one correspondence between the members of a plenum and those of its power-plenum.

The trick of plenum theory is precisely to start by defining a plenum by that presupposition which created the contradiction for sets: the one-to-one correspondence between the members of the collectivity and those of its power-collectivity. What follows is Rescher and Grim's acceptance of whatever (uncomfortable) consequences this "defining mark" of plena has, as long as the consequences still allow for a contradiction-free conception of the totalities in question. If classical logic's Law of Excluded Middle turns out to be a bad bedfellow of the defining mark of plena, so much the worse for classical logic and the Law of Excluded Middle. And this implies that the membership status of some of the members of some of the subcollections of the plenum (in particular, that of some of the members of the diagonalised subplenum) has to remain indeterminate. *This* is the consequence that is too uncomfortable to Kreis:

Das ist aber zweifellos ein unbefriedigendes Ergebnis. Im Rahmen aller gängigen Konzeptionen dessen, was eine Menge ist, ist es Mengen wesentlich, Elemente zu enthalten. Die vorgeschlagene Lösung hat dann aber zur Folge, daß über eine wesentliche Eigenschaft der betreffende Mengen [...] nichts sinnvoll und wahrheitsgemäß ausgesagt werden kann. Man könnte in Erwägung ziehen, daß die Relation des Enthaltenseins für die Mengen der Plenumstheorie, also für Plena und ihre Teilgesamtheiten, gerade **keine** wesentliche Eigenschaft ist. Aber dann würde es unverständlich, in welchem Sinne diese Gesamtheiten überhaupt noch Gesamtheiten sein sollten, worin also der »versammelnde« Charakter der Plena und Sub-Plena bestehen sollte. Der Grundintuition dessen, was eine Menge ist (oder eben eine Gesamtheit, die aber doch wenigstens in einer Familienähnlichkeit zu einer Menge stehen sollte), würde das Fundament entzogen, wenn es für eine Menge (oder eine Gesamtheit) nicht wesentlich sein sollte, Elemente zu enthalten. (pp. 450–1)

Here again, it seems to me, Kreis is judging the proposal from the wrong point of view. Rescher and Grim explicitly admit that, *when seen from the point of view of sets*, plena have surprising, literally "mind-expanding" and "explosive" characteristics. An obvious example is the fact that one can never have even the faintest idea of the size of a plenum (the amount of members it has). Given the

defining mark of plena (a plenum has a member corresponding to each one of its subcollectivities), each subcollectivity, including the plenum itself, will have a corresponding member, but these members will form other subcollectivities, which will have corresponding members, which will ... and so on *ad infinitum*—or maybe we should say *ad nauseam*! In any case, it is clear that any 'stable' conception of the size of a plenum will forever escape our capabilities (Rescher & Grim 2011:66). But this is at most an epistemological predicament. Rescher and Grim's is in principle a realist approach to (the size of) collectivities.

This consideration might help provide an answer to Kreis's objection. Note, first, that it is inaccurate to say that to have elements is not an essential relation for plena, for if we know anything about plena, it is what we know through their defining mark: that *they have an element* corresponding to each one of their subplena, and in the case of membership plena, where the one-to-one correspondence is that of identity, we can say that they *have* all their subplena *as elements*. So the idea of membership, of element containment, is entailed by the most basic notion we have of what plena should be. But note, secondly, that what leads Kreis to affirm that element containment is not an essential relation for plena is that in this case we are not allowed to proceed the way we proceed with sets, namely, by attributing a determinate truth value to sentences of the form ' $x \in C$ ' or ' $x \notin C$ '. But this is at most a "membership classification" problem, an "expressive limitation" (Rescher & Grim 2011:80–1); in other words, it is very much *our* problem, not a problem for plena themselves. If it were retorted that it *is* a problem for plena themselves, for one of them has to be the case (either ' $x \in C$ ' or its negation has to be true), then that would mean that one is not taking at all seriously the rejection of the Law of Excluded Middle, the proposal that there are full gaps in truth value for certain sentences.

But note also, finally, that the 'classification problem' need not be as substantial as it appears to be at first sight. The existence of the diagonalised object does not entitle us to say that element containment is not essential to plena. In effect, if we start by presupposing the existence of a one-to-one correspondence between all subplena of a plenum  $\mathbf{P}$  and all members of the plenum, we find that there must be a subplenum  $\mathfrak{D}$  such that, for any  $w$  in the original plenum  $\mathbf{P}$ , it is indeterminate whether ' $w \in \mathfrak{D}$ ' is true or ' $w \notin \mathfrak{D}$ ' is true. But note that  $\mathfrak{D}$  is not necessarily a plenum. It is a subplenum of  $\mathbf{P}$ , which by no means entails that it is a plenum. So it is clear that the indeterminacy in truth value of the sentences regarding the membership of  $w$  in the subplenum  $\mathfrak{D}$  does not allow us to say anything about whether the property of element containment is or is not essential for plena. In other words, what is indeterminate is the membership status of a member of a subplenum of the plenum  $\mathbf{P}$  (and only in the case of membership plena does this mean indeterminacy of the membership status of the subplenum

$\mathfrak{D}$  with respect to itself), *not* the membership status of the members of the plena with respect to  $\mathbf{P}$ . To say that it is indeterminate whether  $w \in \mathfrak{D}$  or whether  $\mathfrak{D} \in \mathfrak{D}$  by no means implies that it is indeterminate whether  $w \in \mathbf{P}$  or whether  $\mathfrak{D} \in \mathbf{P}$ . So there is no reason why we should not say that *to have elements* is an 'essential' relation of plena. What we could say, at most, is that it is not an essential aspect of a particular subplenum of the plenum *to determinately have (the way sets do) exactly the members it has*. And in some cases (that of membership plena) what we could say is that it is not essential for a member of the plenum to be determinately self-membered. But this is a far cry from saying that element containment is not essential to plena.

Moreover, that the diagonalised object creates even the above-mentioned 'expressive difficulty' should not come up as a surprise, nor should it worry us that much. The diagonalised object, even from the time of Cantor's proof of the larger cardinality of the set of real numbers with respect to the set of natural numbers, and the more general proof of the larger cardinality of any Power-set  $P\alpha$  with respect to the set  $\alpha$ , has always been *systematically defined in order to* create the logical indeterminacy from the assumption of the one-to-one correspondence that would exist if the sets in question are equinumerous. Given the axiom of extensionality that characterises sets—two sets are identical *if and only if* they have the same members—plus classical logic, the indeterminacy of membership ascriptions in the case of sets is unbearable. But again: plena are not sets, their members and subcollectivities need not be sets, the axiom of extensionality need not hold sway over plena, subplena, and members of plena, and, finally, classical logic's Law of Excluded Middle is rejected. There is no reason why the 'expressive difficulty' generated by the concept of an object which is defined with all the intention of provoking a contradiction for the size comparison of sets should be a primordial concern for plenum theory. Only for someone who has decided in advance that all collections have to be like sets is the 'expressive difficulty' represented by the logical indeterminacy of the membership status of the elements of the diagonalised subcollectivity unbearable. The rest of us can coexist quite happily with ontological pluralism for collections.

### Concluding Remarks

Let me conclude by pointing to a solution strategy that might help in answering the final family of paradoxes presented by Kreis. I am referring to those Cantorian paradoxes that are stated without presupposing that the absolute totalities in question (the totality of all objects, of all facts, of all propositions) are *sets*. In Kreis's own formulation of the paradox regarding all objects, cited earlier on, the paradox is derived from the presupposition that there is a one-to-one correlation between all objects and themselves. We have two lists: in the first of

them are all objects, in the second one are again all objects. In the second list there will be interpretations of formal languages. One of those items, call it ' $I_D$ ', is an interpretation under which a predicate  $F$  applies to all and only those interpretations under which  $F$  does not apply to itself. Now, does  $F$  apply to the assumed correlate of  $I_D$ ? Surely, one of the two must be the case: either  $F$  applies to the correlate of  $I_D$  or it does not apply to it—this much is assumed by Kreis. From this assumption, Kreis derives the desired contradiction, for of course  $F$  applies to the correlate of  $I_D$  if and only if it does not apply to it. But this is precisely the point at which calling into question Kreis's assumption might prove useful. Why *must* either ' $F$  applies to the correlate of  $I_D$ ' or its negation be true? We could even proceed in a way parallel to that of Rescher and Grim in proposing plenum theory. We could take for granted the existence of the identity correlation and then get rid of whatever turns out to be an uncomfortable bedfellow of the proposed correlation. If the Law of Excluded Middle is incompatible with the identity of every object with itself, then so much the worse for the Law of Excluded Middle.

Perhaps Kreis would see here a confirmation of his Adorno-inspired 'negative dialectics', for the task of this philosophical project is accomplished by showing that the paradoxes of absolute totalities can only be solved, if at all, by paying a high price, i.e. by abandoning cherished, traditional, general principles of thought. So be it. But note that this would not be the first time in the history of philosophy that the Aristotelian *tertium non datur* would be called into question. Intuitionist logicians reject the general validity of the principle. More importantly: Kreis must surely count himself amongst the cautious when it comes to affirming the truth of the principle, or even its overall respectability throughout the history of philosophy. For, as we saw, he presents a fairly sympathetic interpretation of Kant's transcendental idealism, and according to transcendental idealism thus interpreted certain sentences, namely those about empirically inaccessible objects, lack a truth-value. Thus the currently envisaged solution strategy need not be that unpalatable after all.

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### Notes:

[1] An eloquent passage, which in my opinion nicely sums up Kreis's interpretation of Kant, is the following one: "Erscheinungen sind nach Kant nicht etwa in dem Sinne subjektabhängig, daß sie kausal vom menschlichen Geist hervorgebracht würden und/oder mit geistigen Zuständen numerisch identisch wären

(damit wäre ein *ontologischer* Antirealismus formuliert). Erscheinungen sind aber sehr wohl in dem Sinne subjektabhängig, daß notwendigerweise erkenntnistranszendente Aussagen keinen Wahrheitswert haben (damit ist das spezifisch *Transzendente* von Kants Idealismus erfaßt)" (p. 108).↵

[2] "Weil die Welt der Inbegriff alles dessen ist, was man *in* der Welt anschauen kann, kann die Welt als Ganze nichts sein, das man anschauen kann" (p. 122).↵

[3] Henry Allison criticises Gram's argument, claiming that the Kantian argument can be defended by an appeal to Kant's doctrine of transcendental illusion (Allison 2004:390). A criticism of Allison's argument, on the grounds that Kant is not entitled to an appeal to that doctrine in the present context, can be found in Bravo Zamora (2016:107–9).↵

[4] Cf. Allen Wood's remarks in his debate with Allison, (Wood, Guyer & Allison 2007:7–9).↵

[5] Cf. Williamson (2003), Section IV, especially p. 26.↵

[6] Cf. Plantinga & Grim (1993:277–8), where Grim presents a paradox very similar to the one I'm about to present.↵

[7] Cf. Hallett (1984:7, 28).↵

[8] In a somewhat different context, Marcus Giaquinto provides the following example, which in *my* opinion *partially* explains and supports Cantor's Domain Principle: "Consider the predicate ' $x > 0$ ' and for some  $y$ ,  $y = 1/x$ '. If, in this predicate, the range of both the free variable ' $x$ ' and the bound variable ' $y$ ' is the class of all rational numbers, the predicate expresses the condition of being a positive rational number with a rational inverse, which is satisfied by all positive rationals. If we alter the range of ' $x$ ' to the class of integers while the range of ' $y$ ' remains the class of rationals, the predicate expresses the condition of being a positive integer with a rational inverse, which is satisfied by just the positive integers. Finally, if the range of both ' $x$ ' and ' $y$ ' is the class of integers, the predicate expresses yet another condition: that of being a positive integer with an integer inverse, which is satisfied only by 1. So the condition expressed by a predicate can be sensitive to the ranges of its free and bound variables" (Giaquinto 2002:241n.19). Another example, in this case with the explicit purpose of endorsing Cantor's Domain Principle, is given by Priest (2002:125).↵

[9] Cf. Fraenkel, Bar-Hillel & Levy (1973:118).↵

[10] As Cartwright (1995:7–8) does.↵




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